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Trumpet mouthpiece equivalent lengths

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Abstract

The mouthpiece of a brass instrument serves two functions. On the one hand it provides a supportive interface to the player's lips. But it also controls the tuning of the instrument resonances, which should be harmonically related in order to give the best playing response. This acoustic effect of the mouthpiece is described by its frequency dependent equivalent length *Leq*, which can be defined as the shortest length of cylindrical tubing that could replace the mouthpiece and give the same boundary condition at the junction with the rest of the instrument. For most of the instrument resonances, *Leq* is shorter than 1/8 of the wavelength and the mouthpiece is well described by a two parameter model using the total volume and the frequency of its first or Helmholtz resonance. Any given mouthpiece may need to be tuned to its instrument in order to improve its playing characteristics, exemplified by a crescendo test and by an attack response test as well as by intonation of the various registers. Examples are given for a number of soprano brass instruments.

Keywords: trumpet, mouthpiece, equivalent length

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1 Introduction

The mouthpiece of a brass instrument such as a trumpet is removable and often is purchased separately from the instrument. Thus it is tempting to consider the mouthpiece as independent of the instrument. However, when being played the mouthpiece is in place and forms part of the whole instrument. In particular, the resonance frequencies of the instrument and their relationships can be altered when the mouthpiece is changed, and this can change the instrument's playing properties. This paper describes how a trumpet mouthpiece can be adjusted in order that its effective acoustic length is suitably matched to the rest of the instrument.

2 Background

It happens that most wind instruments, and brass instruments in particular, allow the player to play a series of notes that fit the harmonic series, just on a given length of tubing. These correspond to the natural modes or resonances of the tube. Of course it is usually convenient musically if these notes are harmonic and in tune, but it also turns out to have a large role in how well the instrument plays. Henri Bouasse [1] in 1929 was one of the first to recognize this. Art Benade [2,3] later elaborated this concept and described a series of playing tests that could be used to guide a maker or player to improve the playing behavior of a given instrument. William Cardwell [4,5] devised a method to improve the tuning of brass instrument modes and recognized the importance of the mouthpiece and leadpipe in determining brass instrument tuning. Robert Pyle [6] elucidated the effects of the instrument bell, mouthpiece, and leadpipe and described them in terms of their equivalent lengths.

3 Theory

3.1 Definition of equivalent length

The concept of equivalent length *Leq* assists in the description of the resonance frequencies of a complicated system in terms of the lengths of a simpler system. In this case, a large portion of the tubing in the central section of a trumpet is cylindrical. The *Leq* of the entire instrument or of the bell or the mouthpiece is the equivalent length of cylinder that could replace it and still produce the same resonance frequencies. It is not a single number, but varies with frequency.

A brass instrument is essentially a tube closed at the mouthpiece end and open at the bell end. A cylinder of length L in meters, closed at one end and open at the other, has many resonances or modes whose frequencies f_m are given by equation (1), in which m=1,2,3,… is the mode number and c is the speed of sound in meters/second.

$$f_m = \frac{c\,(2m-1)}{4L} \tag{1}$$

Accordingly, given a tube that does not have uniform cross-section, its equivalent length *Leq* can be determined from the measured resonance frequencies f_m as in equation (2).

$$Leq = \frac{c\,(2m-1)}{4f_m} \tag{2}$$

The trumpet has four major sections: the mouthpiece, the tapered leadpipe, the nearly cylindrical central bore with tuning slides and valves, and the flaring bell. The equivalent length for the complete instrument is the sum of the equivalent lengths of these sections.

3.2 Helmholtz resonator

The Helmholtz resonator is a simple acoustical system consisting of a cavity with a short open neck. The trumpet mouthpiece resembles such a system except that its neck is somewhat long, but the analogy will only be useful for frequencies up to the first resonance anyway. When the cavity volume is increased, the resonance frequency of the Helmholtz resonator falls. When the neck area is increased or the neck length is decreased, the resonance frequency rises. We are going to connect this resonator at its opening to an extended air column. The resonator's equivalent cylinder has a cross-sectional area *A* equal to the inlet area of the connected air column. At low frequencies when the wavelength of sound is much longer than the dimensions of the resonator, only the volume of the resonator is important and not its shape. The equivalent cylinder at low frequencies has cross-sectional area volume equal to the total volume *V* of the Helmholtz resonator, so its equivalent length is as given in equation (3).

$$Leq_{LF} = \frac{V}{A} \tag{3}$$

At the first resonance frequency, the Helmholtz resonance is indistinguishable from a closed-open cylinder at its first mode resonance, for which the cylinder length is equal to one quarter of the wavelength λ, so that its equivalent length is given by equation (4)

$$Leq_{reson} = \frac{c}{4f_1} \tag{4}$$

3.3 Lumped element transmission line

For calculating the response of arbitrary air column shapes, the air column is considered to be a series of short cylindrical segments. For the segment numbered *i*, the length d_i and cross-sectional area A_i are specified. Then, if the pressure *p* and volume flow *u* are known at the terminating end of a segment, their values at the other end of the segment can be calculated with the help of equation (5)

$$\binom{p}{u}_{in} = \begin{bmatrix} \cos kd_i & R_0 \sin kd_i \\ \frac{1}{R_0}\sin kd_i & \cos kd_i \end{bmatrix} \binom{p}{u}_{term} \tag{5}$$

Here $R_0 = \rho c/A_i$ is the wave impedance of segment with ρ as the air density, and $k = 2\pi f/c$ is the wavenumber at frequency *f.* Damping in the air column can be incorporated by adding an imaginary term to *k*. Since the input impedance *Zin* is defined as the ratio of *p* over *u*, one may substitute *Zin* and 1 for *p* and *u* where they appear on both sides of equation 5. Given an air column profile, perhaps for a mouthpiece, equation (5) can be applied successively. Then, at the end of the mouthpiece, equation (6) can be used to calculate the *Leq*.

$$tan(2k\,Leq) = -Im(F)/Re(F), \text{ with } F=(Zin-R_0)/(Zin+R_0) \tag{6}$$

4 Measurements of equivalent length

To measure *Leq* for mouthpieces, a leadpipe was fit to a long section of brass cylinder; their combined length was 1.335 m. A piston containing a microphone on its face was fit into a cylinder the size of the end of the mouthpiece. A chirp signal that encompasses a band of frequencies was played from a loudspeaker into the far open end of the cylinder. The microphone recorded the response of the air column; the spectrum was calculated by Fourier transform, and the frequencies of the response peaks were measured. The piston was withdrawn by 0.010 m and the measurements repeated over a span of 0.15 m.

This cylinder of variable length, closed by the piston, serves in essence as a surrogate for the equivalent length we seek. For each of the first twelve modes of the system, the data was cast in terms of length as a function of peak frequency and fit to a polynomial, so that after measurement of a peak frequency, *Leq* could be directly determined from the table.

The next step was to remove the movable piston and to put a mouthpiece in its place. The mouthpiece cup opening was sealed with soft wax to a plastic plate that held a microphone.

The chirp signal was played and the frequencies of the response peaks measured, from which the *Leq* were calculated from the functions determined in the previous step.

A similar process was used to measure the equivalent length of the entire instrument by inserting the mouthpiece with plate and microphone into a trumpet instead of using the leadpipe and cylinder. Assuming that the intended resonances frequencies would be harmonic, equation (2) was used to calculate nominal values of *Leq*. These were then subtracted from the measured values to arrive at an error estimate, that is, whether the measured *Leq* were too long for some modes and making them too flat, or were too short for some modes and making them sharp.

5 Modelling effects of changes in mouthpiece bore

In order to show how the *Leq* curve of a mouthpiece changes with the shape of the mouthpiece bore, the method of section 3.3 was used to calculate *Leq* for a typical mouthpiece shape and for variations of it, as detailed below:

> Cup size and volume: the diameter of the cup was varied from 0.0162 m to 0.0170 m.
>
> Throat: the diameter of the smallest area was varied from 0.0033 m to 0.0041 m.

6 Results

6.1 Measurements of mouthpieces

Figure (1) shows the results of *Leq* versus frequency measured for four mouthpieces. Mouthpieces A and B are nominally the same model, although their *Leq* curves are slightly different. Each curve for A and B is the average of three different runs and the error bars show their standard deviation. The two curves come together at low frequencies, because the low frequency limit for *Leq* is equal to the total volume divided by the area of the tube at the end of the mouthpiece, as defined by equation (3). For A and B this limit is indicated by a horizontal line at 0.055 m.

Mouthpieces A and B differ in their Helmholtz frequencies, also called *Fpop*, which are 680 Hz and 710 Hz respectively. This frequency is most easily measured by the same apparatus as before, without the leadpipe and cylinder, but with just the end of the mouthpiece open. The Helmholtz frequency fixes a second point on the *Leq* curve where it intersects the curve given by equation (4), as shown. As a result, the curve in the general vicinity of *Fpop* is steeper for A than for B. This means that those modes will play flatter for A than for B, as compared to the lower modes.

Mouthpieces C and D are also nominally the same model but different from A and B. They have a smaller cup volume and therefore less total volume than A and B, and this is reflected in a lower *Leq* at low frequencies. Their *Fpop* is about 730 Hz, similar to each other but slightly higher than for the other two. All else being equal, a higher *Fpop* is consistent with smaller cup volume, just as a blown bottle filled with water plays a higher frequency. These two curves for C and D have lower *Leq* than for B, which means they play at a slightly higher frequency overall, but that is easily compensated by the tuning slide. Otherwise, they are generally parallel to C, meaning that their relative behavior across the range of the instrument is similar.

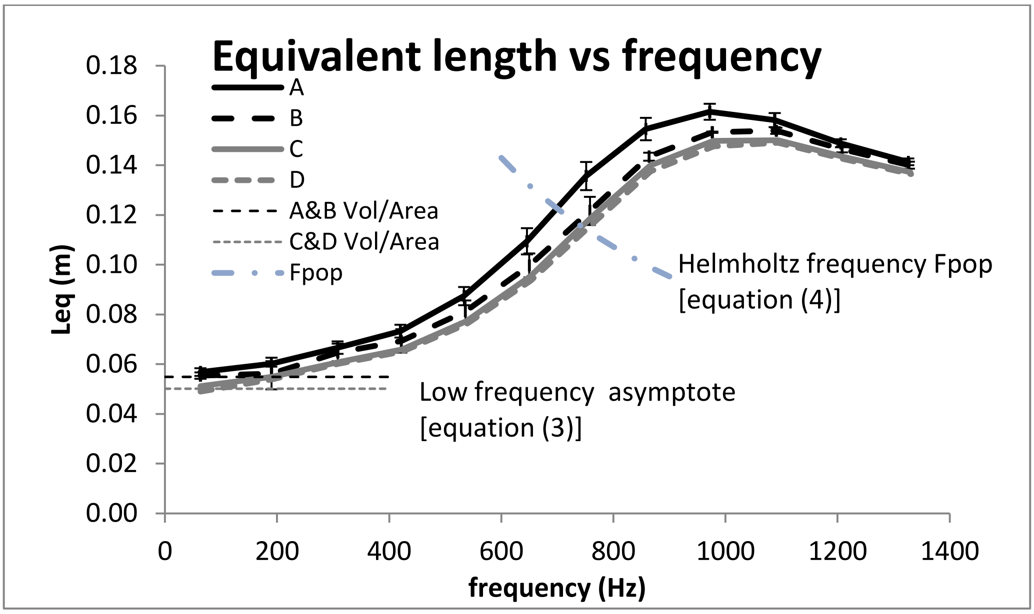

Figure 1 – Equivalent lengths *Leq* of two pairs of trumpet mouthpieces

In figure (2) are shown the *Leq* curves for three mouthpieces of different makes and models. These are generally models with smaller cups than the ones in figure (1), and therefore lower total volume and higher *Fpop*. Some players use these for piccolo trumpets or for high-note playing, which is consistent with the makers' literature. The Helmholtz frequency *Fpop* for the 7E, 7EW, and 12A4a examples measured here are 790, 820, and 745 Hz, respectively, and again these can be found from the intersection of the Leq curves with the curve from equation (4). The 7E curve lies above the 7EW curve at low frequencies, suggesting that it has a higher volume. However, they are otherwise parallel, suggesting that the relative tuning of the mode frequencies is similar.

The third mouthpiece, a 12A4a, has approximately the same Leq at low frequencies as the 7EW, meaning that these two have similar total volumes. However, because the 12A4a has a much lower *Fpop*, its *Leq* curve in the middle of the graph is much steeper, meaning the modes in the range of about 600-1000 Hz (notes E5-C6) will be relatively flat compared to the other two mouthpieces. However, above 1000 Hz the *Leq* curve begins to come back down, causing a sharpening effect for notes above this high C6.

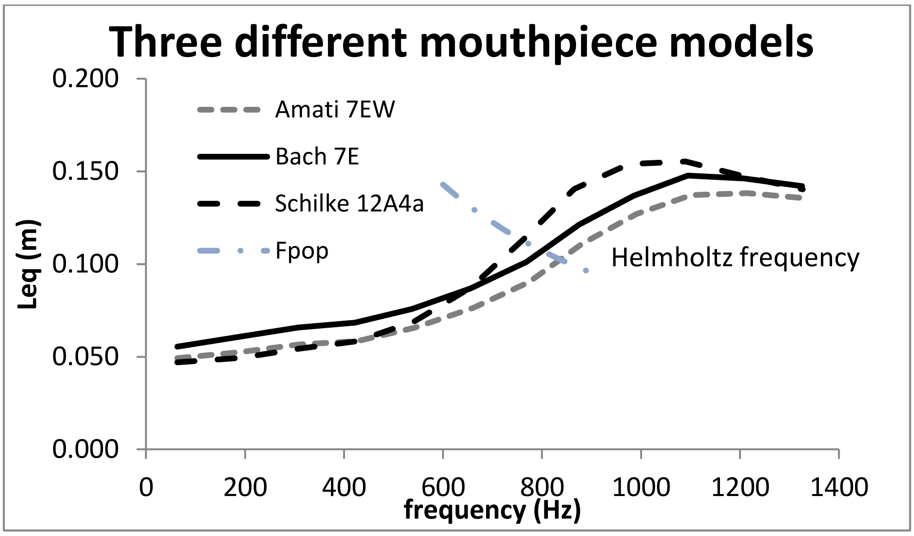

Figure 2 – Equivalent lengths of three different trumpet mouthpieces

6.2 Changing mouthpiece dimensions

Figure (3) shows the results of the method of section 3.3 in which the *Leq* is calculated from a mathematical model. The left panel shows *Leq* versus frequency, similar figures (1) and (2). The right panel shows the relative change in *Leq* as the result of the bore changes. In this figure the cup diameter is being reduced. As a result the total volume will decrease, lowering *Leq* and raising *Fpop*. However, there is also a sharp reduction of *Leq* at frequencies near *Fpop*, which will raise any mode frequencies in that region relative to the others.

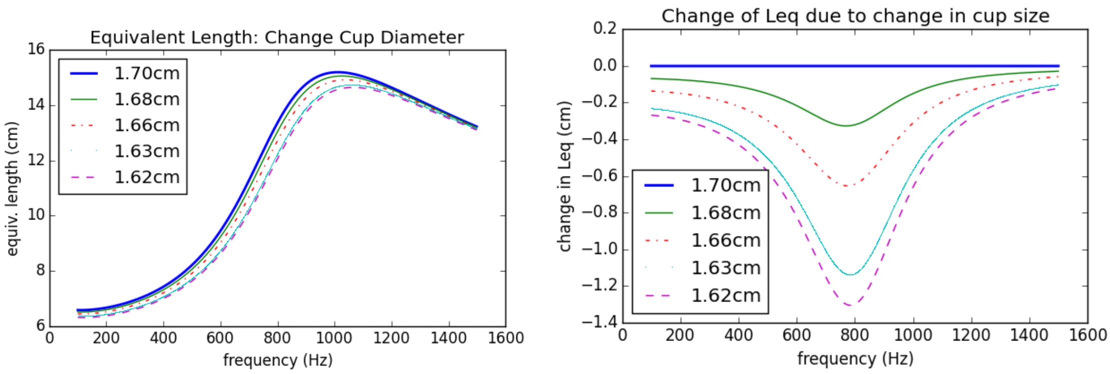

Figure 3 – Effect of changing mouthpiece cup diameter (and volume)

Changing the throat diameter will change the Helmholtz frequency without much change in total volume. This is illustrated in figure (4), which shows the effects of both a larger throat and a smaller one. Again, there is a pronounced effect in the vicinity of *Fpop*.

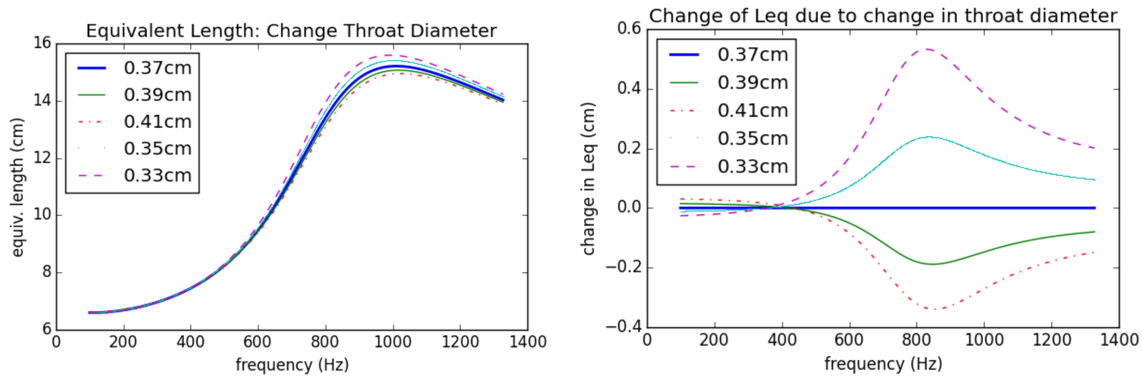

Figure 4 – Effect of changing throat diameter

6.3 Complete instrument

Figure (5) shows the relative tuning errors for the complete instrument and demonstrates the effect that the mouthpiece can have. Higher on the graph means the *Leq* at that frequency is greater and therefore the note will be flatter. This graph has an arbitrary reference point of *Leq*=0 for the low C4 at 230 Hz, which is the second mode of the instrument. This was chosen because a crescendo test on C4 assesses under playing conditions is used to assess whether the mode frequencies are properly aligned into harmonicity.

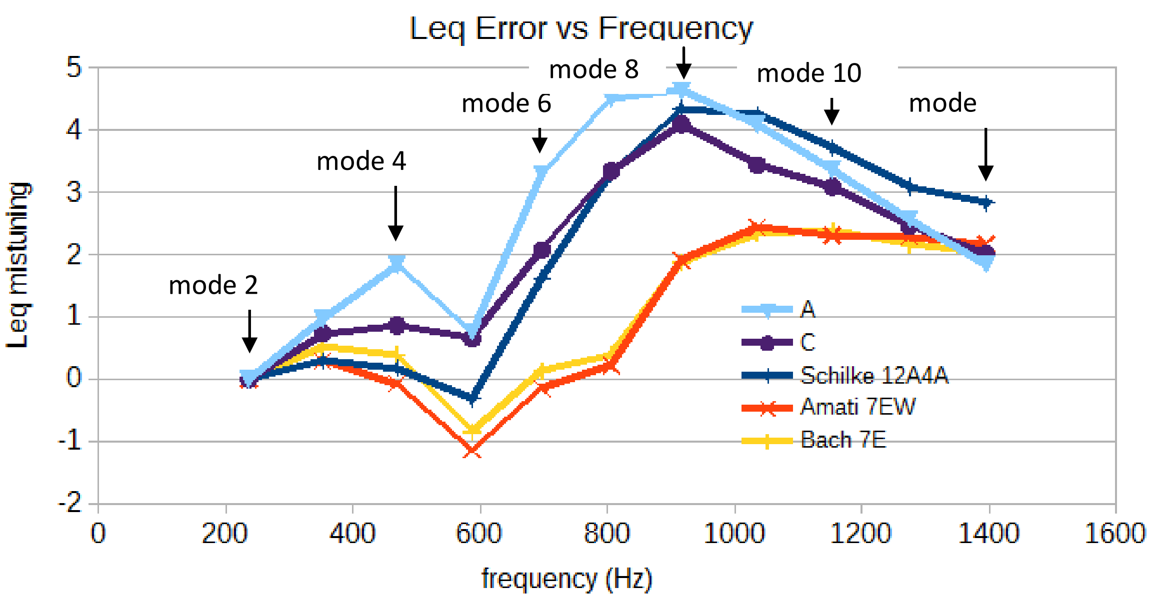

Figure 5 – Leq tuning errors for the complete instrument

7 Conclusions

7.1 Equivalent length measurements and calculations

When mouthpieces do not properly match their instruments, players are not getting all the benefit they might from their expensive investments. In this paper, the concept of equivalent length has been revisited for the purpose of improving this match for the large number of existing instruments for which players might wish to find or modify mouthpieces to suit. Instrument makers sell a wide variety of mouthpieces according to parameters such as cup size, throat size, and backbore shape, but the results in section 6.1 demonstrate that these are not adequate to describe the acoustical performance of the mouthpiece.

In section 6.2 we demonstrate how the acoustical properties of the mouthpiece, and the equivalent length in particular, can be modified. There are other variables that can also be adjusted, such as overall length, the length of the throat, and the taper and flare of the backbore, but they have not been discussed here for space reasons.

It should be noted that in fact the players' lips protrude slightly into the mouthpiece cup, reducing its volume, which would accordingly change its *Leq* in the manner shown in figure (3). This is an important correction routinely applied by musical instrument manufacturers, but to varying degrees.

7.2 Playing tests

Because of the acknowledged discrepancy between acoustic properties of the mouthpiece and instrument when measured without player and when being played, it is important to have a means of making the necessary measurements under playing conditions. One excellent tool is simply to play all of the notes and modes very, very softly, *pianississimo*, for then it will play at the corresponding mode frequency. These notes can be recorded and then analyzed with either a normal tuner or with a spectrum analyzer.

Another tool is the crescendo test, typically done on both the low C4 and on G4. For example, consider the note C4 based on the second mode resonance. The harmonics of this note will fall close to the frequencies of modes 4, 6, 8, 10, and 12. When the note is played softly it will play at the mode 2 frequency. However, when the note is played louder, the higher harmonics will be stronger and their corresponding modes will have greater influence. In figure (5), mouthpiece A shows a mistuning of greater *Leq* at mode 4 than at mode 2, and even more at modes 6 and 8, so the instrument with this mouthpiece will tend to go flat on this crescendo test, which is in fact what happens. On the other hand, one of the 7E or 7EW mouthpieces would be more stable because modes 4 and 6 exhibit very little mistuning, and only the higher modes would cause some disruption. Given the results of section 6.2, a modification for mouthpiece A or C might be designed to improve the harmonicity of the mode frequencies and thereby improve the playability of the instrument with those mouthpieces

With these playing tests and the information provided by the equivalent length measurements and calculations, it is feasible, indeed even routine, to adjust a mouthpiece to match its instrument.

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