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When singing bowls don't sing: a numerical and experimental investigation on the subtle dynamics of Tibetan bowls

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Abstract

Tibetan bowls have been traditionally used for ceremonial or meditation purposes, but also in contemporary music-making. They are handcrafted and produce different tones depending on their shape, size, mass and their alloy composition. Most important is the sound producing technique by impacting and/or rubbing, as well as the excitation location, the hardness and friction characteristics of the excitation stick. In a previous paper, we developed a physically-based method for nonlinear time-domain simulation of Tibetan bowls. Our computational approach, based on a compact modal formulation, produces realistic dynamical responses. In the present paper we focus on an interesting feature of Tibetan bowls: in order to produce self-excited responses, the stick must rub the bowl against the *external* side of the rim, e.g. radially pressing outwards the bowl center. Indeed, experimenting with many bowls showed that they do not sing when rubbed internally. We start documenting this claim with experimental results from representative bowls, and then exploit our computational model in order to reproduce the observed behavior qualitatively. Our results are in good agreement with experiments, thereby demonstrating that internally excited bowls are dissipative and hence unable to sing.

Keywords: Tibetan bowl, friction-induced dynamics, physical-based modelling, modal approach, time-domain simulations

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1 Introduction

Tibetan bowls have been used for centuries as musical instruments and ritual tools, specifically in Buddhist monastics, and various use of these instruments can be today found in contemporary music. They are handcrafted and produce different musically-interesting tones according to their shape, size, mass and their alloy composition. Most important is the sound producing technique which involves the local action of a massive stick, called the puja, and includes both impact and friction excitations, thereby resulting in a combination of ringing and singing tones. Sound emergence then comprises complex nonlinear phenomena usually challenging to address by numerical analysis [1]. From the modelling standpoint, due to the time-vayring excitation contact location of the puja on the bowl, the problem also shares some fundamental concepts peculiar to moving-load dynamical problems [2], which are very common in mechanical engineering applications.
In previous work by Inacio et al. [3], dynamic simulations of Tibetan bowls were presented and a throrough analysis of the physical mechanisms leading to sound production was proposed. Our efforts aimed at describing the physics as closely as possible, in order to provide a full account of the global dynamics, therefore developing a physical model and simulating the general trends observed in playing practice. Our computational approach, based on a compact modal formulation, produced realistic vibratory responses - see reference [3], and provides an efficient tool for closely examinating the subtleties of friction-induced axisymmetric shells vibrations. Besides the number of remarkably stable periodic regimes which may be triggered, an interesting dynamical feature of Tibetan bowls, surely blurred by the unusual playing technique, is the nonexistence of steady-state musical regime when rubbed internally, e.g. by pressing radially outwards the bowl center. Indeed, experimenting with many bowls shows that they do not sing for internal excitation.
In this paper, we document this claim with both experimental and numerical results. To that end, a number of representative Tibetan bowls of different size and mass has been investigated experimentally for different rubbing conditions, and time-domain simulations have been performed by exploiting our computational model which is also briefly presented here. The simulation results are in qualitative agreement with the experiments, and demonstrate that internally excited bowls are dissipative and hence unable to sing.

2 Experiments

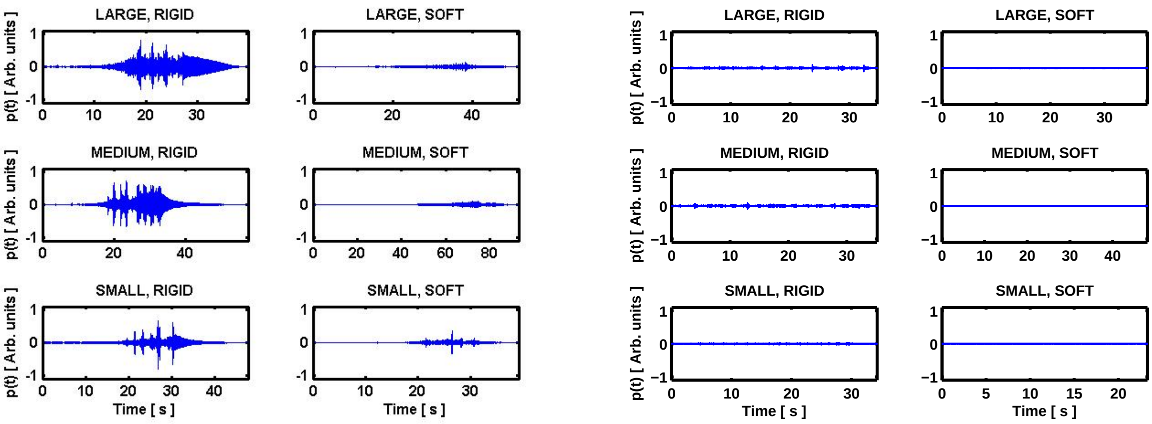
Usually, when Tibetan bowls are rubbed, self-excited regimes settle after several seconds of continuous excitation of the puja stick on the rim. For given initial conditions of the tangential velocity and normal force applied by the musician on the puja, the motion builds up progressively, with an exponential increase of the bowl vibration amplitude followed by nonlinear saturation, and reaches a steady-state regime whose dynamical features strongly depend on the characteristics of the bowl/puja friction and contact interaction. As any nonlinear self-excited oscillators, various dynamical regimes are thus obtained by changing the set of initial conditions, and playing experience shows particularly that periodic regimes of different fundamental frequencies may emerge. Other characteristic feature of bowl sound is the superimposed beating behaviour, perceived by a fixed listener.

Figure 1: Singing Tibetan bowl and puja sticks used in the experiments.

A number of three Tibetan bowls of different size and mass, were played with pujas made of different materials, namely rubber and wood - see Fig. 1. Either external or internal rubbing excitations were attempted, and acoustic response signals were recorded by a microphone, at a distance of about 1 m and an angle of 45° with the horizontal plane. The pressure waveforms obtained for the 12 studied configurations are plotted in Fig. 2. As seen by comparing results in Figs. 2a and 2b, while self-excited regimes were always reached for external excitation, it has been impossible to produce a periodic regime by exciting internally the bowl. Because of the mildly-controlled human playing, several attempts were performed by changing somehow the normal force and/or the travelling velocity of the puja, and other bowls, not documented here, were also tested, and always the same behaviour was present. Physically, this suggests that energy transfers occuring in the system are strongly influenced by the direction of the excitation, which can radically affects the global energy balance. Contrary to externally friction-excited bowls, bowls excited internally manage to absorb all the energy supply provided continuously by the puja. Figure 2a also evidences the superimposed beating behaviour already mentionned. In [3], the origin of the beating is discussed, and surprisingly, it is not related with the geometrical imperfections of the bowl structure, but is a manifestation of the radiation pattern of Tibetan bowl when listeners are fixed in space, caused by the revolution of the linearly unstable modes which follows the puja motion at the spinning frequency.

3 Computational model

A computational model for simulating the dynamics of Tibetan bowls has been proposed by Inacio et al. [3]. It involves the dynamics of a flexible ring coupled to a rigid puja, and



(a) External excitation. (b) internal excitation.

Figure 2: Waveforms of the pressure recorded for the 12 studied configurations. Small to large Tibetan bowls were played, using either soft (rubber-like) or rigid (wood) puja. Recording level is identical for all cases.

subjected to both impact and friction excitations. The model, based on the modal approach, successfully reproduces the singing and ringing responses of the bowls when impacting or rubbing its rim externally. Here, we remind the essential ingredients of the modelling and present the dynamical modal equations. Similarly to [3], the formulation is written in a fixed coordinate frame for the bowl, and refinements in the modelling are included by accounting for the actual moving load on the bowl rim. As observed in other mechanical systems such as pipes conveying flow [4], the influence of moving load can be important for a fundamental understanding of the detailed dynamics. However, knowing that typical spinning frequency of the puja stick is small compared to the frequency of the vibratory motions, one might expect only a small influence of these terms for the problem at hand.

3.1 Dynamics of the bowl

Tibetan bowls are essentially axisymmetric shells, and as any structure of this type, they exhibit normal modes in orthogonal pairs, with near-identical modal frequencies. When excited, the vibratory response of the bowl wall $\mathbf{Y}(\theta,t)$ involves the dynamical coupling of the radial $Y^R(\theta,t)$ and tangential $Y^T(\theta,t)$ motions, and is given by:

$$\mathbf{Y}(\theta,t) = Y^R(\theta,t)\,\mathbf{e_r} + Y^T(\theta,t)\,\mathbf{e_t} \tag{1}$$

where $\mathbf{e_r}$ and $\mathbf{e_t}$ are the unit vectors defining the radial and tangential direction respectively (see Fig. 3). Adopting a modal framework for describing the dynamics, any physical bowl motion can be expressed by modal superposition of the modal amplitudes and respective mode shapes considered for the modal basis. For accurate modelling, the two orthogonal modal families, called A and B, have to be accounted, and the radial and tangential motions are then

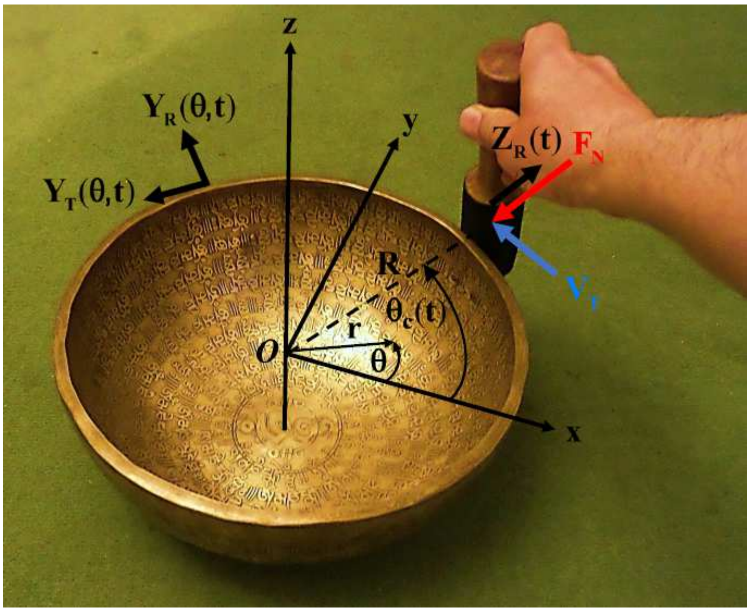


Figure 3: Tibetan bowl excited by tangential rubbing of a massive puja.

formulated as:

$$Y^R(\theta,t) = \sum_{n=1}^{N} \left(q_n^A(t)\, \phi_{Rn}^A(\theta) + q_n^B(t)\, \phi_{Rn}^B(\theta) \right), \qquad Y^T(\theta,t) = \sum_{n=1}^{N} \left(q_n^A(t)\, \phi_{Tn}^A(\theta) + q_n^B(t)\, \phi_{Tn}^B(\theta) \right) \tag{2}$$

where $q_n^{A,B}$ are the modal amplitudes of the two modal families, and $\phi_{Rn}^{A,B}$ and $\phi_{Tn}^{A,B}$ are the corresponding mode shapes in the radial and tangential directions given by:

$$\phi_{Rn}^A(\theta) = \cos(n\theta), \quad \phi_{Tn}^A(\theta) = -\sin(n\theta)/n, \tag{3a}$$

$$\phi_{Rn}^B(\theta) = \sin(n\theta), \quad \phi_{Tn}^B(\theta) = \cos(n\theta)/n \tag{3b}$$

Following Inacio et al. [3], we may then built a dynamical model for the Tibetan bowl, formulated as a set of 2N modal equations which takes the form:

$$m_n^A\, \ddot{q}_n^A(t) + c_n^A\, \dot{q}_n^A(t) + k_n^A q_n^A(t) = f_n^A(t) \tag{4a}$$

$$m_n^B\, \ddot{q}_n^B(t) + c_n^B\, \dot{q}_n^B(t) + k_n^B q_n^B(t) = f_n^B(t) \tag{4b}$$

where $m_n^{A,B}$, $c_n^{A,B}$, and $k_n^{A,B}$ are the modal parameters of the modal families A and B, respectively given by:

$$m_n^{A,B} = \int_0^{\ell} \rho_\ell(\theta) \left\| \phi_n^{A,B}(\theta) \right\|^2 d\ell, \quad k_n^{A,B} = m_n^{A,B} (\omega_n^{A,B})^2, \quad c_n^{A,B} = 2 m_n^{A,B} \omega_n^{A,B} \zeta_n^{A,B} \tag{5}$$

with ℓ the bowl perimeter and ρ_ℓ the linear mass density. The modal excitations $f_n^{A,B}$ in Eq. (4) stem from the projection of the external force $f(x,t)$ on the respective modal basis, computed as

$$f_n^{A,B}(t) = \int_0^{\ell} f(x,t) \phi_n^{A,B}(x) d\ell \tag{6}$$

Since the sound producing technique involves impact and/or rubbing around the bowl rim, both nonlinear impact and friction interaction are accounted in our modelling. Excitations will be assumed as a travelling point-force of the form

$$\mathbf{f_p}(\theta_c(t),t) = \left[f^R(t)\,\mathbf{e_r} + f^T(t)\,\mathbf{e_t}\right]\delta(\theta-\theta_c(t)) \tag{7}$$

where $\theta_c(t) = \Omega t$ is the contact angle and $\Omega = \mathrm{v_T}/R$ is the spinning velocity of the puja, with $\mathrm{v_T}$ its tangential velocity imposed by the musician, and R the bowl radius. Substituting Eq. (7) in (6), the modal forces are expressed in the form of:

$$f_n^{A,B}(t) = f^R(t)\,\phi_{Rn}^{A,B}(\theta_c(t)) + f^T(t)\,\phi_{Tn}^{A,B}(\theta_c(t)) \tag{8}$$

3.2 Interaction forces

3.2.1 Radial interaction

The radial force exerted by the puja stick on the bowl at any angular location $\theta_c(t)$ can be simply described by a penalty formulation such as:

$$\begin{cases} f^R(t) &= -K_c\Big(Y^R(\theta_c,t)-Z(\theta_c,t)\Big)-C_c\Big(\dot{Y}^R(\theta_c,t)-\dot{Z}(\theta_c,t)\Big) & \text{for contact} \\ f^R(t) &= 0 & \text{for non-contact} \end{cases} \tag{9}$$

where K_c and C_c are two penalty constants referred to as the contact stiffness and damping coefficient, and which are directly related to the puja material. From Eq. (9), it can be seen that the radial force $f^R(t)$ is negative for an external excitation since contact occurs when $Y^R(\theta_c,t) > Z(\theta_c,t)$, resulting in an opposite positive force on the puja. For internal excitation, contact occurs for $Z(\theta_c,t) > Y^R(\theta_c,t)$, and the reverse sign applies for the radial force.
For accurate modelling of the interaction, it must be reminded that equations are here formulated in a fixed coordinate frame for the bowl, with a moving load acting on it, so that the motion of the bowl fells by the puja differs than if the coupling location was fixed. One consequence is that any time derivative of the bowl motion appearing in the formulations of the bowl/puja interaction must be expressed using material derivatives as:

$$\frac{dY(\theta(t))}{dt} = \frac{\partial Y}{\partial t} + \mathrm{v_T}\frac{\partial Y}{\partial \theta} \tag{10}$$

with $\mathrm{v_T}$ the constant speed of the moving puja. Bowl velocities in Eq. (9) are then computed as:

$$\left.\frac{dY^R(\theta(t))}{dt}\right|_{\theta_c} = \sum_{n=1}^{N}\left[\dot{q}_n^A(t)\,\phi_{Rn}^A(\theta_c) + \dot{q}_n^B(t)\,\phi_{Rn}^B(\theta_c)\right] + \mathrm{v_T}\sum_{n=1}^{N}\left[q_n^A(t)\left.\frac{\partial\phi_{Rn}^A}{\partial\theta}\right|_{\theta_c} + q_n^B(t)\left.\frac{\partial\phi_{Rn}^B}{\partial\theta}\right|_{\theta_c}\right] \tag{11}$$

For the problem at hand, however, one expects a small influence of the second term in the right-hand-side of Eqs. (10) and (11) since normal playing uses low values of $\mathrm{v_T}$, less than 1 m.s^{-1}, so that the spinning frequency $\Omega = \mathrm{v_T}/R$ remains small compared to the frequencies involved in bowl vibrations.

3.2.2 Friction interaction

It was found that contrary to bowed strings, friction-excited bars and shells are systems which are sliding for most of the time [5]. For synthesis purpose, this is a very convenient because the simulations of stick-slip regimes is usually tricky numerically, and small time-step are required to capture the sudden stick-slip transitions when solving the equations through explicit numerical schemes. Inacio et al. [3] implemented a simple approach for computing the highly non-linear friction interaction force, and used a regularized friction law which seems adequate for simulating singing Tibetan bowls. Retaining the same approach here, the friction force is formulated as:

$$\begin{cases} f^T(t) = -\mu_d\big(\dot{Y}_c(\theta_c,t)\big)\,|f^R(t)|\,\mathrm{sign}\big(\dot{Y}_c(\theta_c,t)\big) & \text{if} \quad |\dot{Y}_c(\theta_c,t)| \geq \varepsilon \quad \textit{sliding} \quad (12a) \\ f^T(t) = -\mu_S\,|f^R(t)|\,\dot{Y}_c(\theta_c,t)/\varepsilon & \text{if} \quad |\dot{Y}_c(\theta_c,t)| < \varepsilon \quad \textit{pseudo-adherence} \quad (12b) \end{cases}$$

where $\dot{Y}_c(\theta_c,t) = \mathrm{v_T} - \dot{Y}^T(\theta_c,t)$ is the bowl/puja relative velocity, μ_S is the static friction coefficient, and μ_d is the dynamic friction coefficient used during sliding, which depends on the relative bowl/puja tangential velocity as:

$$\mu_d(\dot{Y}_c) = \mu_D + (\mu_S - \mu_D)e^{-|\dot{Y}_c|/\mathrm{v_0}} \tag{13}$$

where μ_D is an asymptotic limit of the friction coefficient when $|\dot{Y}_c| \to \infty$ and $\mathrm{v_0}$ is a parameter controlling the decay rate of the friction coefficient with $\dot{Y}_c(\theta_c,t)$. The friction model (13) can be readily fitted to typical experimental data by adjusting the empirical constants μ_S, μ_D and $\mathrm{v_0}$. According to Inacio et al. [3], a regularization parameter ε of about 10^{-4} $\mathrm{ms^{-1}}$ seems adequate for reproducing realistic behaviour of rubbed Tibetan bowls. Again, note that $\dot{Y}_c$ used in (12)-(13) must be computed through Eq. (11).

3.3 Dynamics of the Puja

The model for the excitation has now to be completed by writing the puja dynamics. The puja is modeled as a rigid body of mass m, neglecting any elastic dynamics. Its radial motion $Z(\theta_c,t)$ in the direction normal to the bowl surface (see Fig. 3), is simply governed by the rigid-body dynamical equation:

$$m\ddot{Z}(\theta_c,t) = \pm f_N - f^R(\theta_c,t) \tag{14}$$

where f_N is a constant normal force imposed by the player, and $f^R(\theta_c,t)$ is the radial component of the puja/bowl interaction forces. Note that here either external or internal excitations are considered. The normal force f_N will be negative for an external excitation while it will be positive for internal excitation.

3.4 Fully-coupled computational model

Grouping Eqs. (4), (9), (12) and (14), the self-sustained response of the coupled system is formulated as a set of $2N+1$ second-order modal ODEs, written in the matrix form as:

$$\begin{pmatrix} \mathbf{M_A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M_B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & m \end{pmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_\mathbf{A} \\ \ddot{\mathbf{q}}_\mathbf{B} \\ \ddot{Z} \end{Bmatrix} + \begin{pmatrix} \mathbf{C_A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C_B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{Bmatrix} \dot{\mathbf{q}}_\mathbf{A} \\ \dot{\mathbf{q}}_\mathbf{B} \\ \dot{Z} \end{Bmatrix} + \begin{pmatrix} \mathbf{K_A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{Bmatrix} \mathbf{q_A} \\ \mathbf{q_B} \\ Z \end{Bmatrix} = \begin{Bmatrix} \mathbf{f_A}(t) \\ \mathbf{f_B}(t) \\ f_Z(t) \end{Bmatrix} \tag{15}$$

where the column vectors $\mathbf{q}_{\mathbf{A},\mathbf{B}}(t)$ represent the modal displacements for the two orthogonal modal families, and $\mathbf{f}_{\mathbf{A},\mathbf{B}}$ and $\mathbf{f}_{\mathbf{Z}}$ are column vectors stemming from the modal projection of the forces applied to the bowl rim and the puja respectively, written in a compact form as:

$$\mathbf{f}_{\mathbf{A}}(t) = f^T(t)\left[\phi^A_{T1}(\theta_c),\ldots,\phi^A_{TN}(\theta_c)\right]^T + f^R(t)\left[\phi^A_{R1}(\theta_c),\ldots,\phi^A_{RN}(\theta_c)\right]^T \tag{16a}$$

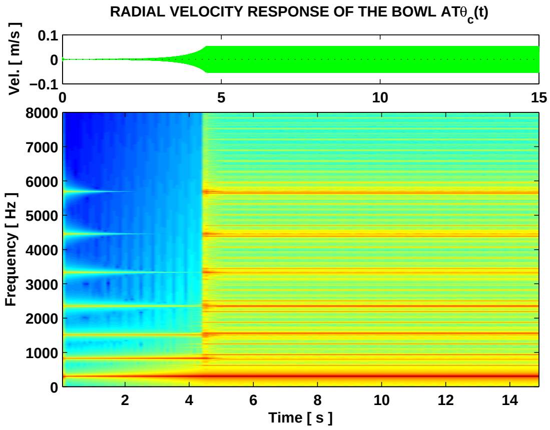
$$\mathbf{f}_{\mathbf{B}}(t) = f^T(t)\left[\phi^B_{T1}(\theta_c),\ldots,\phi^B_{TN}(\theta_c)\right]^T + f^R(t)\left[\phi^B_{R1}(\theta_c),\ldots,\phi^B_{RN}(\theta_c)\right]^T \tag{16b}$$

$$f_Z(t) = \pm f_N - f^R(t) \tag{16c}$$

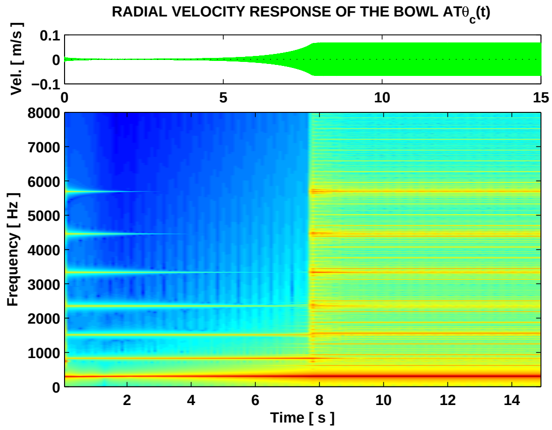
From Eq. (16), it can be seen that the modal forces stemming from the travelling puja fully couple the 2N+1 degrees of freedom of our system.

4 Numerical simulations

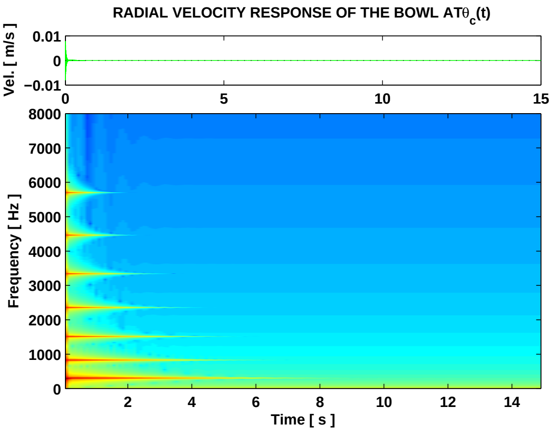
Numerical computations were performed for reproducing the experimental observations, by changing the direction of the constant normal force excitation, as well as for different assumptions concerning the contact and friction characteristics of the bowl/puja interaction. The case of softer and stiffer pujas were considered by testing contact stiffnesses K_c of 10^5 N/m and 10^6 N/m, with friction parameters of $\mu_S = 0.8$, $\mu_D = 0.4$, $\mathrm{v}_0 = 0.1$ and $\mu_S = 0.4$, $\mu_D = 0.2$, $\mathrm{v}_0 = 0.1$ respectively. Numerical integration of the closed set of nonlinear coupled equations (15) was achieved using a discrete version of the direct integration method [6, 7]. In order to cope with the large settling times that arise with singing bowl, 15 seconds of computed data were generated, enough to accomodate transients for external excitation, and modal equations were solved using a safe time-step of 10^{-6} s. The simulations were based on modal data identified experimentally on a typical Tibetan bowl, which can be found in [3], assuming 7 flexible modes for the bowl modal basis. The puja is modelled as a simple mass of 20 g, moving at a tangential velocity of $\mathrm{v}_\mathrm{T} = 0.3$ m.$^{-1}$, and a constant normal force $F_N = \pm 3$ N is assumed (positive for external excitation and negative for internal excitation).
Fig. 4 shows the time-history and corresponding spectrogram of the bowl radial velocity, which mostly controls the sound radiation, for four tested configurations. Results in Fig. 4a and 4b were obtained while rubbing the bowl externally with the soft and rigid puja respectively, and those of Fig. 4c and 4c pertain to an internal excitation for similar conditions. As seen, singing regimes are always reached for external excitation, while no self-excited motion is obtained when exciting internally the bowl, so that simulated results support the qualitative observations presented in Sec. 2. The computational model appears appropriate for studying such subtle dynamical features, and some light can be shed by examining the contributions of the conservative and nonconservatives forces in the energy balance. Finally, Figure 5 is a plot of the bowl velocity at the contact point, together with the effect of the convective load on both radial and tangential velocity components - see Eq. (11). The first comment concerns the 3-order difference in amplitude between the velocity values, which highlights the very small influence of the convective load on the global dynamics, and confirms the assumptions used in [3]. Also, notice that its effect is more important on the radial than tangential components of the bowl velocity, which is coherent with the analytic expressions of the mode shapes derivatives.



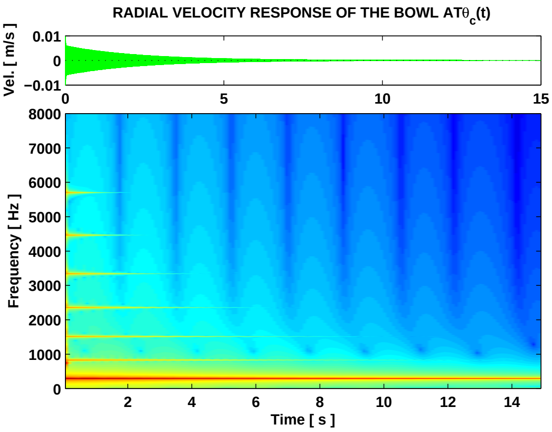
(a) Soft puja.



(b) Stiff puja.



(c) Soft puja.



(d) Stiff puja.

Figure 4: Radial velocity time-history and corresponding spectrogram for external (top) and internal (bottom) rubbing excitation.

5 Conclusions

In this work, we started to document a striking feature of the dynamics of Tibetan singing bowls as they do not sing when rubbed internally. This was first demonstrated simply by playing experience, testing a representative set of Tibetan bowls with different pujas. The nonexistence of stable musical regimes was also illustrated by nonlinear time-domain simulations of a physical model, which thereby corroborate the experimental observations. Greater understanding of the dissipative behaviour of internally-excited Tibetan bowls will be addressed in future work, based on energy concerns.

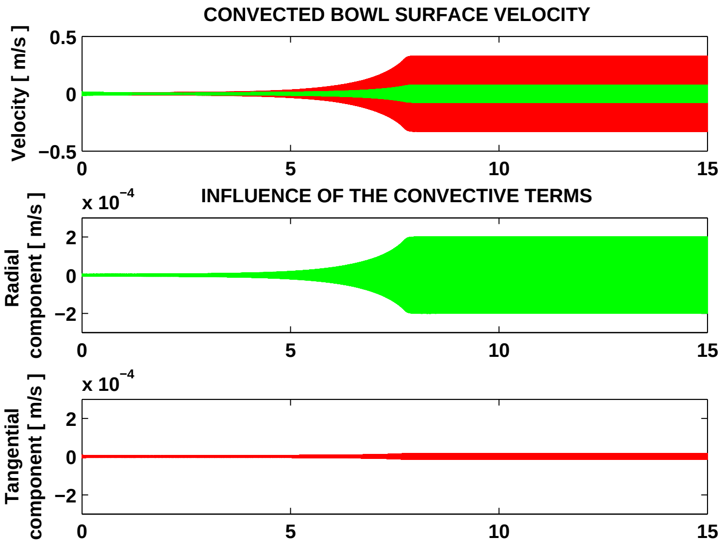


Figure 5: Tangential (red) and radial (green) bowl surface velocity at the contact point, and respective influence of the terms corresponding to the convected load - see Eq.(11).

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