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# Mode switching in an air-jet instrument

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## Abstract

It is well known that air-jet instruments, such as the recorder, undergo transitions between oscillation modes as a function of the blowing pressure. For example, the recorder undergoes a transition from its fundamental mode of frequency *f* to the mode an octave higher (2*f*) as the blowing pressure is increased. In theoretical studies of the recorder the interaction of the air jet with the labium and feedback from the resonator tube to the air jet play central roles in creating a stable oscillation and in determining the blowing pressure at which this transition occurs. However, the details of these interactions, especially the feedback from the resonator tube, are not completely understood. We use Navier-Stokes-based simulations of the recorder to study these interactions for the recorder. We find, in accord with reports of musicians, that the transition blowing pressure is reduced when the resonator tube cross-section is reduced. Movies of the air jet dynamics are used to probe the origin of this effect. By studying a hypothetical recorder with a square resonator tube, we are able to separate the effects of the air jet Reynolds number and sound pressure amplitude.

**Keywords:** recorder, mode switching, Navier-Stokes simulations

# MODE SWITCHING IN AN AIR-JET INSTRUMENT

## 1 Introduction

When a wind instrument, such as a recorder, is played with a small to moderate blowing speed, a musical tone at the fundamental frequency *f* of the instrument is produced. As the blowing speed is increased there is a transition to the mode an octave higher with frequency approximately 2*f*. The blowing speed at which this transition occurs is of great interest to a player and instrument maker, since it determines the dynamic range available for any particular note.

It is known to recorder players that mode switching for bass recorders occurs at a relatively lower blowing speed than is the case for treble or soprano recorders, making it more difficult to play bass instruments in their lowest register [1]. The reason why bass recorders behave differently than other instruments in the recorder family is not understood. In this work Navier-Stokes-based simulations are used to investigate the reason for this difference.

## 2 Modelling mode switching in a wind instrument

Figure 1a shows a schematic of a recorder. A player (not shown) blows from the left into a channel producing an air jet. Upon exiting the channel the air jet is directed toward a sharp edge (the labium). An air jet created in this way is extremely sensitive to external perturbations. The interaction of the air jet with the labium causes the air jet to undergo oscillations in which it moves alternately above and below the labium. If the resonator tube were not present the frequency of this oscillation is controlled by hydrodynamic feedback from the labium to the air jet, producing what is known as an edge tone [2]. In isolation, the air jet-labium system would oscillate at a frequency that is proportional to the speed of the jet as it emerges from the channel.

The proximity of the channel and labium to the resonator in a real recorder (Fig. 1a) causes the oscillations of the jet to produce acoustic waves in the tube. The resonator tube has a high Q, yielding a large acoustic pressure at its resonant frequencies and strong feedback to the air jet at those frequencies. This feedback causes the jet oscillation to “lock onto” a frequency close to one of the resonant frequencies of the resonator, and maintain that frequency for a range of blowing speeds. If the blowing speed is increased sufficiently, the oscillation can jump to a new frequency close to one of the other resonant frequencies of the resonator tube. This mode switching has a strong influence on the playability of the instrument and has been the subject of several studies [3-5].

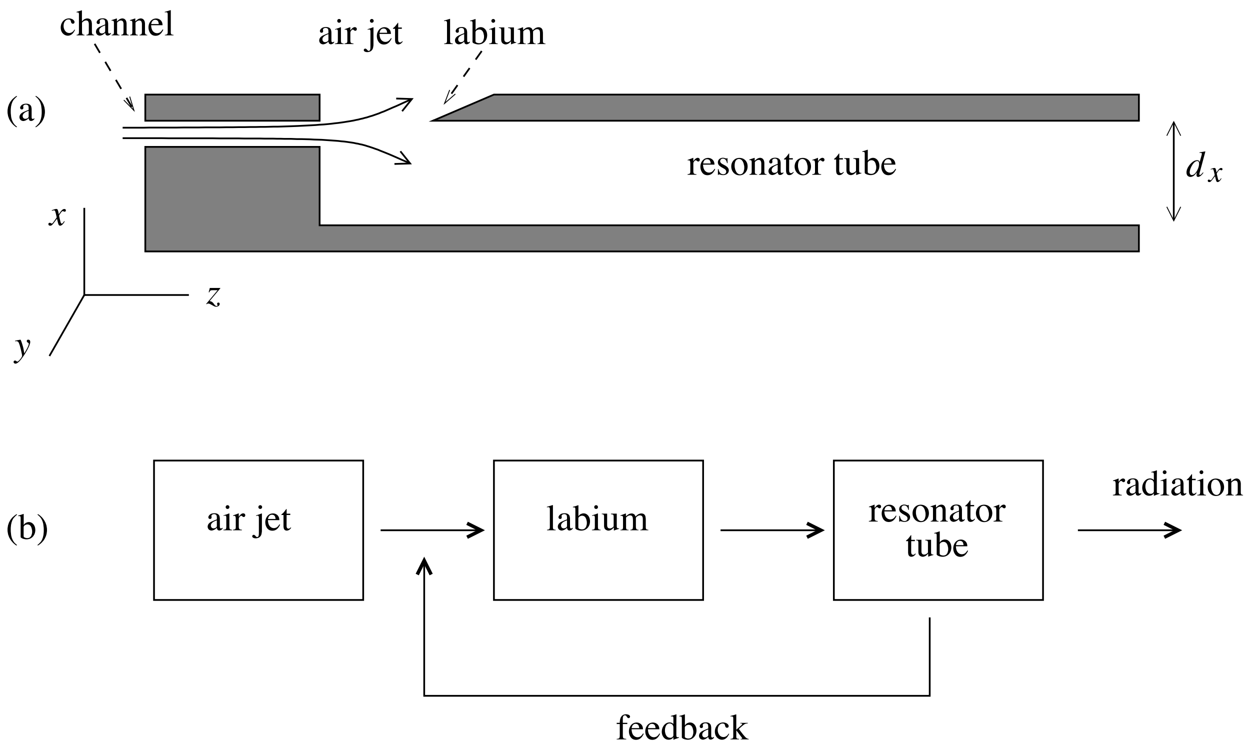


**Figure 1: (a) Schematic of air flow in the neighbourhood of the labium of a recorder. (b) Lumped model of a recorder. Feedback from the resonator tube locks the frequency of air jet oscillations to one of the resonant modes of the tube.**

Modelling studies of this mode switching have generally been based on lumped models of the recorder like the one shown in Fig. 1b. The general picture of an oscillator coupled to a resonator with feedback applies to many instruments, so the lumped model in Fig. 1b is very similar to lumped models for other instruments (see, e.g., [6]). Models of this kind have proven to be extremely useful in understanding a variety of problems but they have limitations, since certain key parameters or relationships are usually needed so that a lumped model can make testable predictions. In most cases, these parameters and relationships must be estimated from qualitative arguments or from experimental studies. For example, the lumped model of the recorder in Fig. 1b requires some sort of estimate of the magnitude, phase, and nature of the feedback to the air jet, as this feedback plays a crucial role in determining the conditions for stable oscillation and the thresholds for mode switching. Empirical models of this feedback have been developed and given useful insights [3], but a fundamental understanding is still of great interest. In this paper we have taken a different approach in which Navier-Stokes-based modelling is used to investigate mode switching. One of the specific goals of our work is to obtain a first-principles quantitative picture of this feedback and thereby understand why mode switching is different for bass as compared to treble and soprano recorders.

# 3 Method

The compressible Navier-Stokes equations were solved numerically using an explicit finite difference-time domain algorithm described elsewhere [7]. The recorder model was similar to the one described in [7]. The resonator tube had length $L$ = 105 mm, and a rectangular cross section of height $d_x$ (see Fig. 1a) and width $d_y$ (in the direction perpendicular to the plane of Fig. 1a), and the behaviour was studied for three different recorder geometries with different values of $d_x$ and $d_y$ as listed in Table 1. The channel had a uniform cross section of height of 1.0 mm and width $d_y$, the distance from the end of the channel to the labium was 4.0 mm, and the labium angle was $\theta \approx 12^o$; these values are typical of real recorders. The Navier-Stokes equations were solved on a non-uniform Cartesian grid. Inside and near the recorder the grid spacing was typically in 0.1 mm in the plane of Fig. 1a and 0.1 - 0.5 mm perpendicular to the plane, and the time step was 0.1 μs (as set by the Courant condition). The recorder was contained in a closed rectangular box with dimensions 60x60x300 $mm^3$ and with a total of approximately $2 \times 10^7$ grid points. Other details of the simulations are given in [7].

**Table 1: Three recorder geometries studied in this work.**

| Recorder geometry | $d_x$ (cm) | $d_y$ (cm) | $L$ (cm) |
|---|---|---|---|
| 1 | 10 | 10 | 105 |
| 2 | 5 | 10 | 105 |
| 3 | 10 | 5 | 105 |

A very useful comparison of different instruments in the recorder family has been given in [8]. The data compiled for a number of different recorders reveals that the dimensions of the instruments scale differently when normalized by the acoustic length of the instrument. The acoustic length is defined as the inverse of the wavelength of the lowest available note, and is proportional to the inverse frequency of that note. An interesting difference between the bass and other recorders is that the scaled diameter of the resonator tube for the sopranino instrument is more than 30% greater than the scaled diameter of the bass recorder. In other words, the resonator tube of the bass recorder has a smaller cross section than one would expect if all dimensions were scaled the same as in the sopranino and soprano recorders. This prompted us to carry out a series of simulations with hypothetical recorder geometries in which the cross sectional area of the resonator tube is varied by changing $d_y$ and $d_z$, with other dimensions held fixed (Table 1). In contrast to the round (or approximately round) resonator tube cross sections found in actual recorders, our studies of tubes with rectangular cross sections allows us to separate the effects of changes in the two transverse dimensions, which may provide additional clues into the physical origin of differences in the behaviour.

# 4 Results

Typical results for the sound pressure outside the recorder as a function of time, *p(t)*, are given in Fig. 2. These results are for recorder #1 in Table 1, which has a resonator tube with $d_x$ = 10 cm and $d_y$ = 10 cm, and at two different blowing speeds *u.* (Here and below the blowing speed *u* is the speed at the center of the air jet as it emerges from the channel.) At the lower blowing speed of *u* = 30 m/s (Fig. 2a) there is an initial transient period lasting about 10 ms, after which *p(t)* exhibits a steady oscillation. At the higher blowing speed of *u* = 45 m/s (Fig. 2b) a steady oscillation is seen after about 15 m/s, but with a higher frequency. In both cases the behaviour seen between 20-30 ms persists to indefinitely longer times.

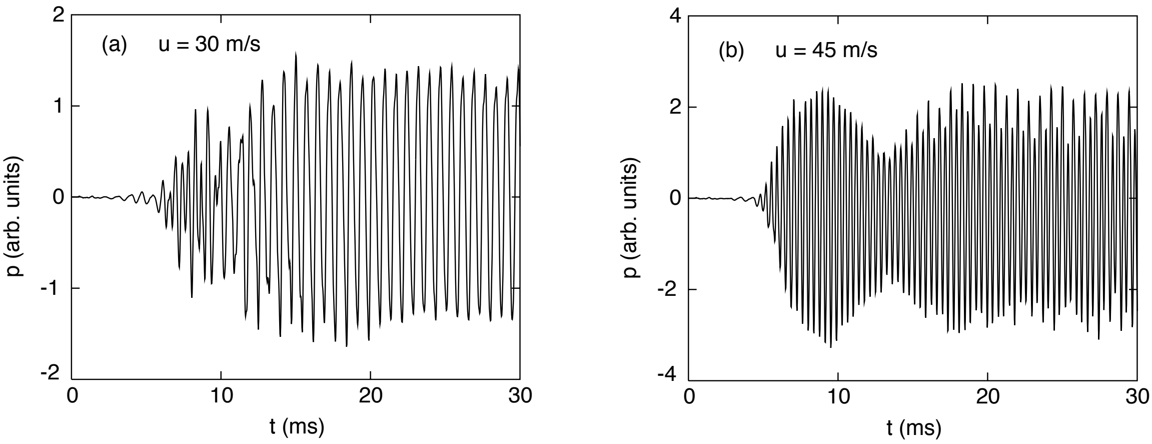


**Figure 2: Results for the sound pressure as a function of time for recorder #1, which has a square resonator cross section with $d_x$ = $d_y$ = 10 cm. (a) Blowing speed *u* = 30 m/s. The recorder oscillates in its fundametal model. (b) Blowing speed *u* = 45 m/s. The oscillation mode is now the second harmonic.**

Figure 3a shows spectra for the tones in Fig. 2. At the lower blowing speed the tone is dominated by a component at the fundamental frequency $f_1$. Harmonics are visible at $2f_1$, $3f_1$, and $4f_1$, but the power at these frequencies is smaller than that at the fundamental by two orders of magnitude or more. At the higher blowing speed of *u* = 45 m/s, the strongest component is now at $2f_1$, which is larger than the fundamental component by more than an order of magnitude. Figure 3b shows the peak power at the fundamental ($P_1$) and at the second harmonic ($P_2$) as a function of blowing speed. Regime change, that is, a change from a tone dominated by the component at $f_1$ to a tone for which the second harmonic at $2f_1$ is largest, occurs between blowing speeds of 40 and 45 m/s.

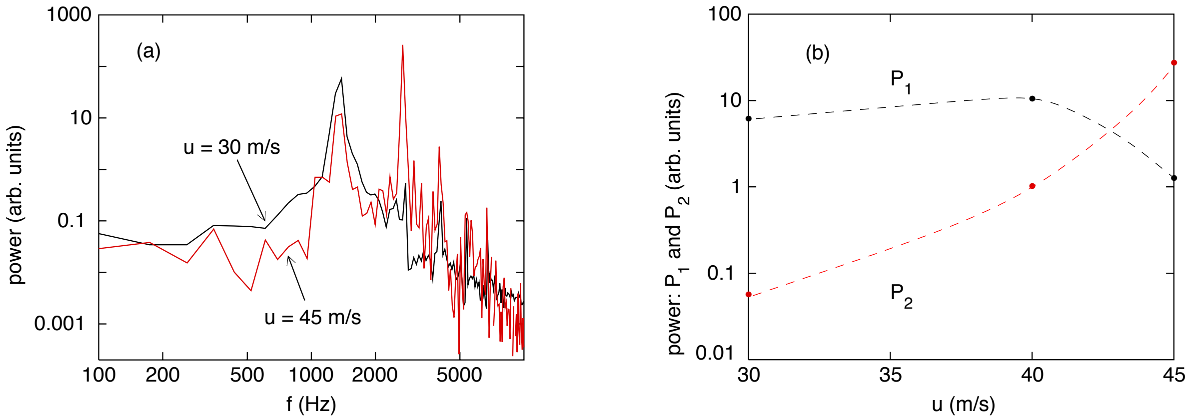


**Figure 3: Results for recorder #1 which has a square cross section with $d_x = d_y$ = 10 cm. (a) Power spectra of the tones *p(t)* in Fig. 2. (b) Peak power $P_1$ at the fundamental frequency ($f_1$) and peak power $P_2$ at the second harmonic ($2f_1$) as a function of the blowing speed.**

The results in Fig. 3b show that regime change for this recorder geometry, which has a resonator tube with a square cross section, occurs at a blowing speed near 43 m/s. Changing the resonator cross section causes regime change to occur at a different blowing speed. Figure 4a shows spectra for recorder #2 in Table 1, which has a cross section $d_x$ = 5 cm, $d_y$ = 10 cm, and recorder #3 which has $d_x$ = 10 cm, $d_y$ = 5 cm (Fig. 4b). In both cases the mode at the second harmonic is dominant when $u$ = 40 m/s, indicating that regime change occurs at a lower blowing speed than found with recorder #1 (Fig. 3b).

The thresholds for regime change for the three different resonator geometries are shown in Fig. 5. These geometries differ only in their resonator cross sections. All other dimensions, including the overall length, the channel height and length, and the channel-labium distance are the same. Figure 5 shows that reducing the cross section reduces the threshold for regime change.

We noted earlier that observations of regime change in real instruments indicate that it is more difficult to play bass instruments in their lowest register, i.e., without having the tone jump by an octave [1]. This amounts to saying that regime change occurs more easily, i.e. at a lower blowing speed in a bass instrument. We noted above that the ratio of the diameter of the resonator to its length is smaller in bass recorder than in other recorders [8]. This experimental observation concerning regime change in bass recorders is thus in good accord with the findings in Fig. 5, that regime change occurs at a lower blowing speed with a smaller resonator cross section when the overall resonator length is held fixed. An interesting feature of our results is that the threshold for regime change does not depend simply on the cross sectional area. Recorders 2 and 3 have the same cross sectional areas, but their thresholds for regime change are quite different.

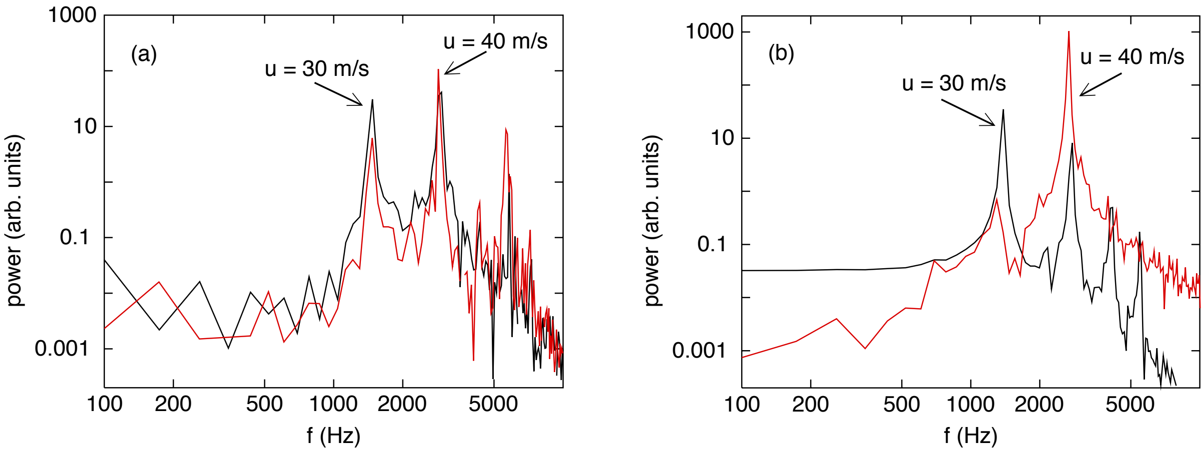


**Figure 4: Spectra for recorders #2 and #3 in Table 1, each at two different blowing speeds. (a) Recorder #2, which has a resonator with cross section $d_x$ = 5 cm and $d_y$ = 10 cm. Regime change occurs for $u$ less than 30 m/s. (b) Recorder #3, which has a resonator with cross section $d_x$ = 10 cm and $d_y$ = 5 cm. Regime change for this recorder occurs at a blowing speed between 30 and 40 m/s.**

The blowing speed at which regime change occurs depends on the feedback from the resonator to the air jet, as indicated schematically in Fig. 1b. We next consider what we can learn about that feedback connection by examining the pattern of air flow near the labium found in our simulations.

Figure 6 shows snapshots of the flow pattern near the labium for the three recorder geometries we have investigated. All of these flow patterns were obtained with the same blowing speed ($u$ = 40 m/s). In Fig. 6a, with recorder #1 which has a square resonator cross section ($d_x = d_y$) the recorder was in mode #1, while for the cases in Fig. 6b and 6c the recorder was in mode #2 (the second harmonic). It is not obvious how best to compare these three cases, since different modes are dominant. In Fig. 6 we have chosen to compare at times in each oscillation cycle at which the air jet is very close to horizontal as it emerges from the channel (i.e., parallel to the axis of the channel) and then bending upward as it approaches the labium. At this moment in the oscillation cycle and for all three recorder geometries we find a general clockwise vortex located beneath the air jet and that extends roughly to the bottom of the resonator tube. This vortex is "weakest," that is, least pronounced for recorder #1 (Fig. 6a), is strongest for recorder #2 (Fig. 6b), and is intermediate in strength for recorder #3 (Fig. 6c). Note that in Fig. 6b this vortex seems to push considerably more flow into the resonator than in the other two cases, suggesting a stronger coupling between the air jet and the resonator. This is also the recorder geometry at which regime change occurs at the lowest blowing speed.

The flow patterns in Fig. 6 were all recorded at a particular time in each oscillation cycle, so one might wonder how or if the appearance of this vortex changes significantly during each cycle. The answer seems to be that while the vortex pattern does change somewhat in appearance during the course of the oscillation, the general result described above does not. At all times there is a clockwise vortex beneath the air jet, and this vortex is always weakest in recorder #1 (the square resonator) and strongest in recorder #2.

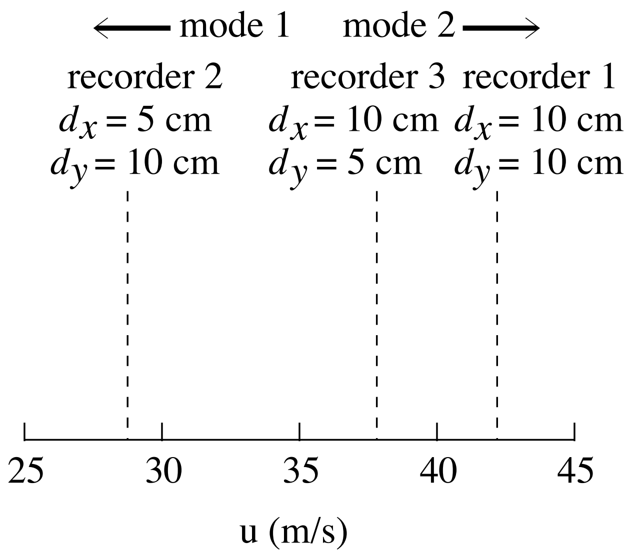


**Figure 5: Threshold blowing speed *u* for regime change for the three recorder geometries considered in Fig. 3b and Fig. 4. Mode #1 is at the fundamental frequency $f_1$, while mode #2 is an octave higher ($2f_1$). Regime change occurs sooner (at a lower blowing speed) when the resonator cross section is reduced. The smallest threshold for regime change is found for recorder #2, which has the smallest dimension in the *x* direction in Fig. 1a.**

At present, our analysis of the flow patterns in Fig. 6 is highly qualitative. With that caveat, we suggest that the vortex beneath the air jet in Fig. 6b is affected by being constrained by the smaller tube dimension in the vertical (*x*) direction in the figure, and that this strengthens the feedback to the air jet causing regime change at the lower blowing pressure. This mechanism should not be present in recorder #3 (Fig. 6c), in which the resonator tube has the same dimension along *x* as in Fig. 6a. In recorder #3, which has a smaller suppression of the threshold for regime change, the motion in the *y* direction (perpendicular to the plane of the figure) must play a role. We suspect that there may also be significant vortex motion in the *y-z* plane beneath the air jet but more simulations will be needed to confirm if this is the case. We should add that when comparing the behavior of the three recorders at the same blowing speed, the Reynolds number of flow in the channel is the same, but the net flow is greater in the resonator of recorder #2 since its cross section is smaller relative to the channel cross section than in the other recorders.

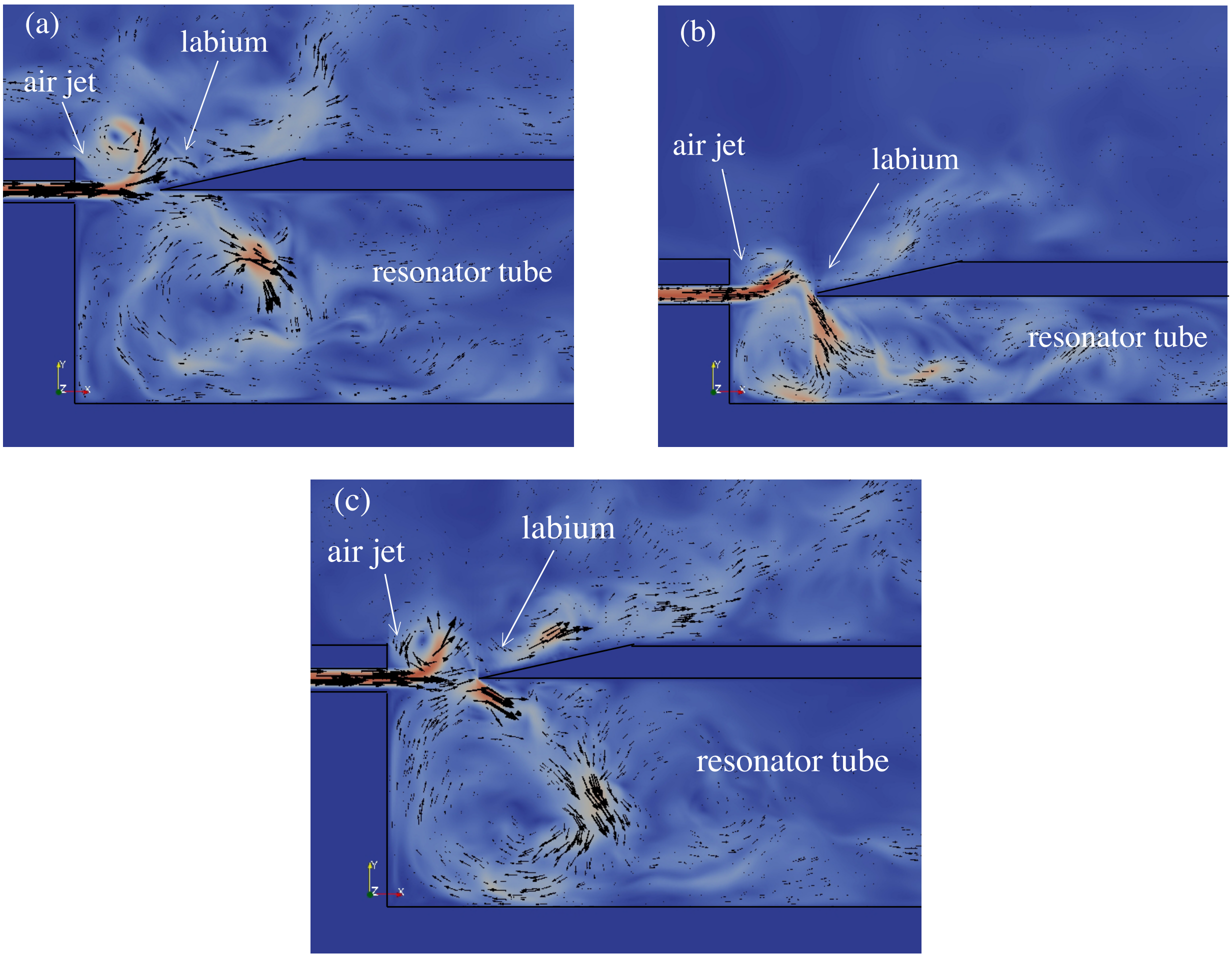


**Figure 6: Patterns of air flow near the labium with a blowing speed of $u$ = 40 m/s for the three different recorder geometries: (a) Recorder #1 with $dx = d_y$ = 10 cm, (b) Recorder #2 with $d_x$ = 5 cm and $d_y$ = 10 cm, (c) Recorder #3 with $d_x$ = 10 cm and $d_y$ = 5 cm. The solid black lines outline the surfaces of the recorder, including the channel. The air jet travels from left to right in the channel, with the black arrows including the air velocity. The blue to red color in the background indicates the magnitude of the velocity at each point. These figures show only the region near the channel and labium. The resonator tube extends far beyond the right edges of these figures.**

# 5 Summary

The Navier-Stokes equations have been used to simulate air flow through recorders of various dimensions to study the mechanisms that determine regime change as the blowing conditions are varied. We find, in agreement with the findings of musicians, that reducing the cross sectional area of the resonator tube while keeping its length fixed reduces the threshold for regime change from the fundamental mode to the mode an octave higher. Our simulations suggest that a vortex forms inside the resonator and beneath the air jet, and that the interaction of this vortex with the walls of the resonator play an important role in the feedback that determines the conditions for regime change. More detailed and quantitative analyses of our flow results should yield further insight into this behaviour.

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